Integration of GNSS and INS

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Aided INS

To limit the drift, an INS is usually aided by other sensors that provide direct measurements of the integrated quantities. Examples of aiding sensors:

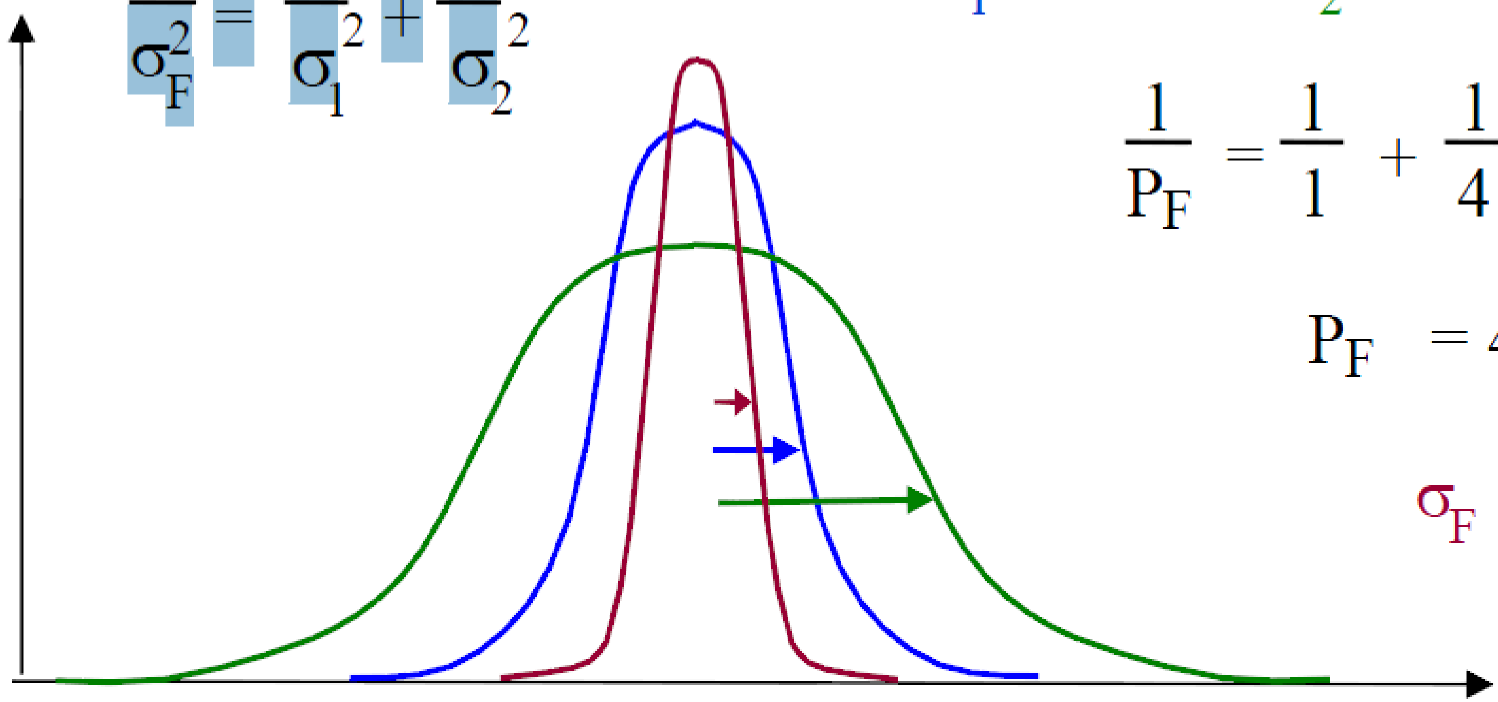
Sensor:	Measurement:
Pressure meter	Depth/height
Magnetic compass	Heading
Doppler velocity log	$\boldsymbol{v}_{\underline{E}B}^{B}$ (or $\boldsymbol{v}_{\underline{W}B}^{B}$, $\underline{w}$ater)
Underwater transponders	Range from known position
GPS	$\boldsymbol{p}_{EB}^{E}$
GPS (Doppler shift)	$\boldsymbol{v}_{\underline{E}B}^{E}$
Multi-antenna GPS	Orientation

Uncertainty reduction - Information gain from entropy reduction

$$\frac{1}{P_F} = \frac{1}{P_1} + \frac{1}{P_2} \qquad P_F = \frac{P_1 P_2}{P_1 + P_2}$$

$$\frac{1}{\sigma_F^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

$\sigma_1 = 1$ $\quad$ $\sigma_2 = 2$



$$\frac{1}{P_F} = \frac{1}{1} + \frac{1}{4} = \frac{5}{4}$$

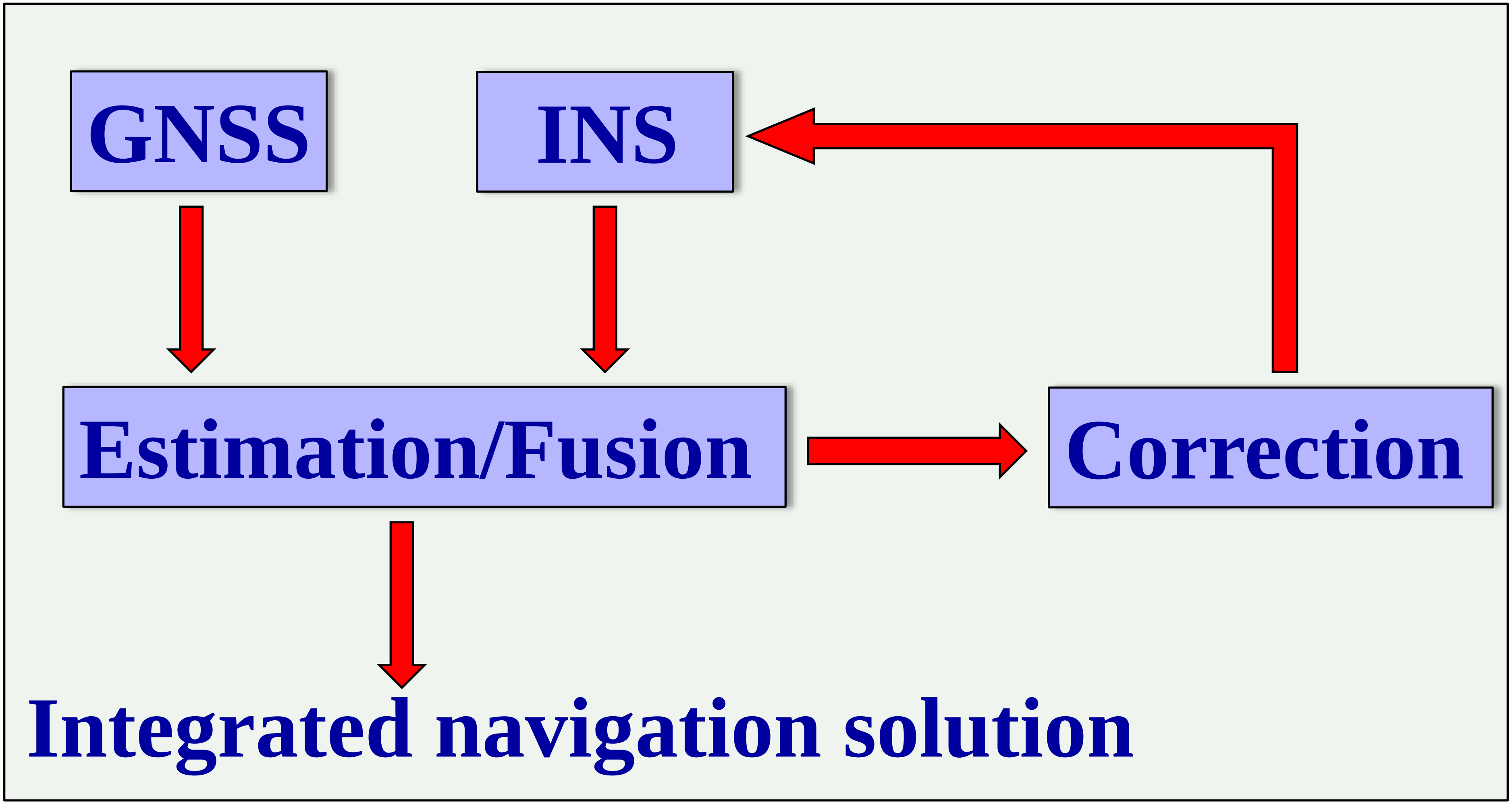
$$P_F = 4/5$$

$$\sigma_F = 0.90$$

INS/GNSS characteristics

Characteristics	INS	GNSS
Accuracy of navigational solution	Good short term accuracy which deteriorates with time	Good long term accuracy but noisy in short term
Initial conditions	Initial alignment	Not required
Attitude information	Available	Typically not available
Sensitive to gravity	Yes	No
Self-contained	Yes	No
Jamming immunity	High	No
Output data rate	High	Low

Architecture of integrated system

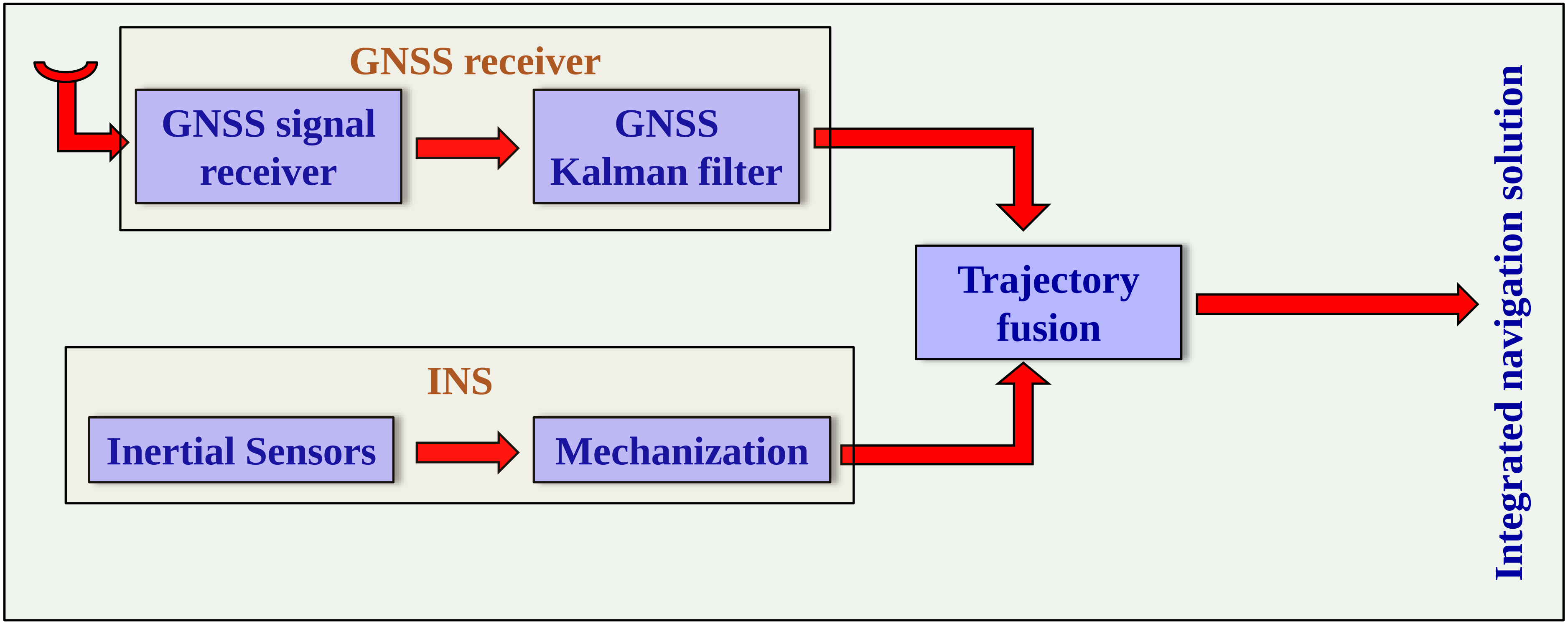


Types of INS/GNSS Integration

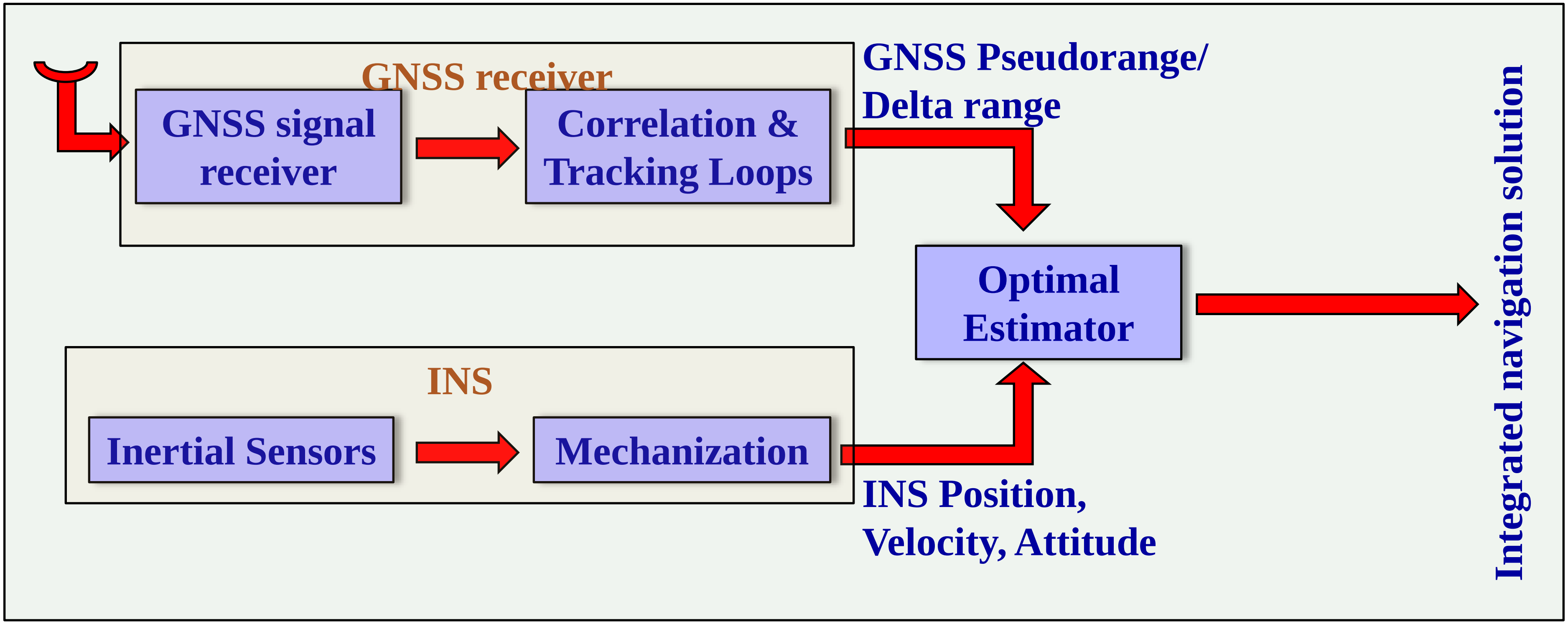
Different forms of INS/GNSS integration architectures have been proposed to attain maximum advantage depending upon the type of use and the degree of simplicity versus robustness. The three main integration architectures are:

1. ***Loosely coupled.***
2. ***Tightly coupled.***
3. ***Ultra-tightly or deeply coupled.***

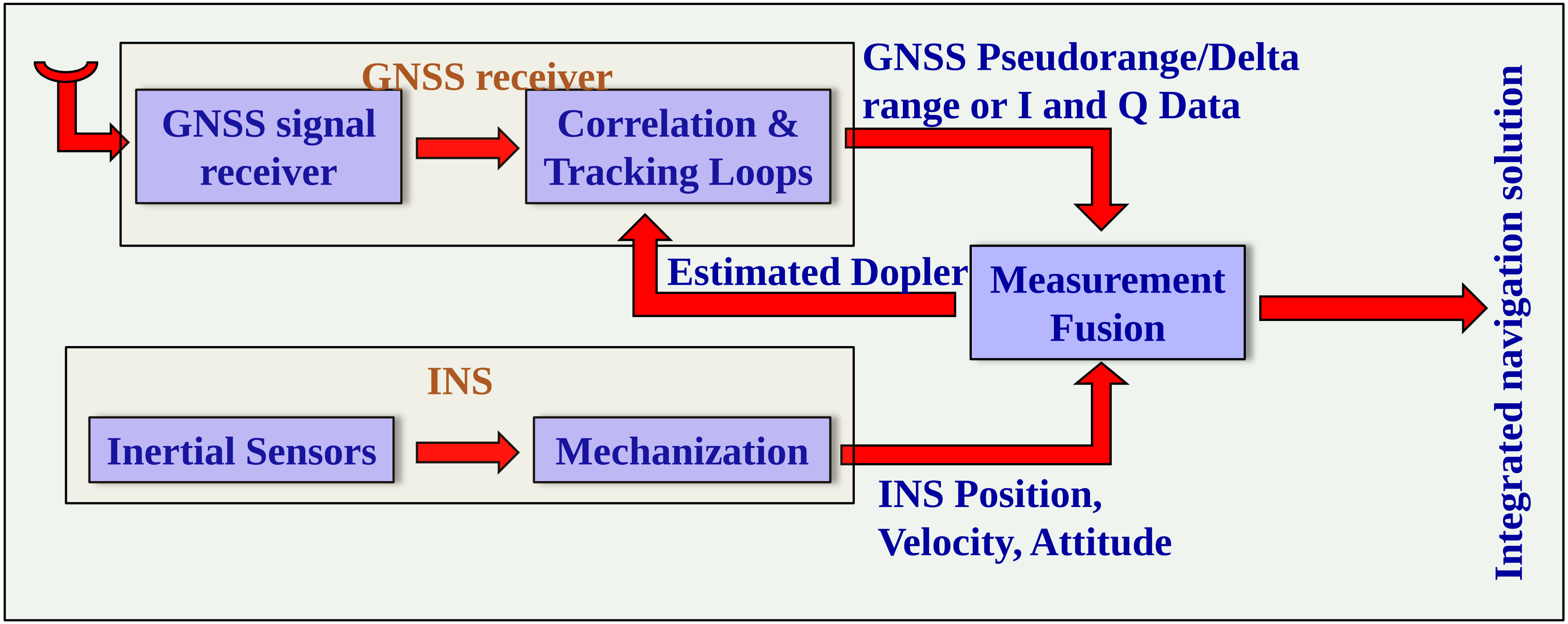
Loosely Coupled INS/GNSS Integration



Tightly Coupled INS/GNSS Integration



Ultra-Tightly Coupled INS/GNSS Integration



INS/GNSS Fusion Algorithm

There are several algorithms for optimal fusion of GNSS and INS data, the major ones being various forms of Kalman filter (KF), particle filter (PF) and artificial intelligence (AI). Traditionally, Kalman filtering has been the method of choice for fusing navigational information from various sources. It is an optimal recursive algorithm (Maybeck 1979) which processes all of the available measurements, regardless of their precision, to optimally estimate the current value of the state of interest and, importantly, also furnishes the uncertainty of its estimate.

The INS and the aiding sensors usually have complementary characteristics. To merge data from sensors many different methods can be applied. One of the most robust approach is Kalman filter. Direct combination of measurements from two or more sensors in Kalman filter complicates its structure and increses state vector. To avoid this most often the error state Kalman filter is used. The differences of all redundant information serve as its measurements. This can be done by running navigation equations on the IMU-data, and compare the outputs with the corresponding aiding sensors.

Kalman filter

Probably the most common optimal filtering technique is that developed by Kalman (1960) for estimating the state of a linear system. Kalman filter can stated as follow: Given our knowledge of the behavior of the system, and given our measurements, what is the best estimate of position and velocity?

$$x(k+1) = F(k)x(k) + G(k)u(k) + v(k)$$

$$z(k) = H(k)x(k) + w(k)$$

$x(k)$	***State vector***
$F(k)$	***Transition matrix***
$G(k)$	***Control matrix (i.e., mapping control to state variables).***
$u(k)$	***Control vector***
$v(k)$	***Gaussian white noise with process variance matrix (i.e., error due to process). Q :***

$$Q(k) = E[v(k)v(k)']$$

$z(k)$	***Measurement variables.***
$H(k)$	***Measurement matrix (i.e., mapping measurements onto state).***

$w(k)$ ***Measurement noise with Measurement variance matrix (i.e., error from measurements):***

$$R(k) = E[w(k)w(k)']$$

P : State variance matrix (i.e., error of estimation).
K : Kalman gain.
Linear description saves Gaussian distribution of state vector and measurement vector. Such type of system is called Gauss-Markovian system.

Solution

Prediction step

$$\hat{x}(k+1|k) = F(k)\hat{x}(k|k) + G(k)u(k)$$

Update step

$$\hat{z}(k+1|k) = H(k)\hat{x}(k+1|k)$$

$$v(k+1) = z(k+1) - \hat{z}(k+1|k)$$

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + W(k+1)v(k+1)$$

$$P(k+1|k) = F(k)P(k|k)F(k)' + Q(k)$$

$$S(k+1) = H(k+1)P(k+1|k)H(k+1)' + R(k)$$

$$W(k+1) = P(k+1|k)H(k+1)'S(k+1)^{-1}$$

$$P(k+1|k+1) = P(k+1|k) - W(k+1)S(k+1)W(k+1)'$$

Extended Kalman Filter

In real life very small number of systems can be regarded linear. Usually they are non-linear, non-Gaussian, or the noise is non-additive. INS are typical examle of non-linear system. Extended Kalman filter is proposed to deal with non-linear systems. If even only one from the equations below is non-linear the system is regarded non-linear. The general desription looks like:

$$x(k+1) = f[k, x(k), u(k), v(k)]$$

$$z(k) = h[k, x(k), w(k)]$$

EKF

In the case of INS, the noise can be regarded additive Gaussian:

$$x(k+1) = f[k, x(k), u(k)] + v(k)$$

$$z(k) = h[k, x(k)] + w(k)$$

The solution of this system can be found appling EKF. Often EKF is called suboptimal (Kalman filter is optimal for linear systems). This is elegant euphemism to avoid sayng non-optimal.

EKF

The main difference between Kalman and extended Kalman filters is in the transition matrix. In the case of EKF transition matrix is approximated by Jacobian of transition matrix F (from Taylor series).

$$\mathcal{J}(f) = \begin{bmatrix} \frac{\partial f_1(x,u)}{\partial x_1} & \cdots & \frac{\partial f_1(x,u)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m(x,u)}{\partial x_1} & \cdots & \frac{\partial f_m(x,u)}{\partial x_n} \end{bmatrix}$$

EKF

Prediction step

1. State vector:

$$\hat{x}(k+1|k) = f[k, \hat{x}(k|k), u(k)] + v(k)$$

2. Covariance matrix of state vector

$$P(k+1|k) = F(k)P(k|k)F^T(k) + Q(k)$$

Here $F(k) = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}(k|k),u(k)}$ $H(k+1) = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}(k+1|k)}$

EKF

Update step

1. Innovation:

$$\tilde{y}(k+1) = z(k+1) - h[\hat{x}(k+1|k)] + v(k)$$

2. Covariance innovation matrix

$$S(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1)$$

3. Gain

$$K(k+1) = P(k+1|k)H^T(k+1)S^{-1}(k+1)$$

4. State vector

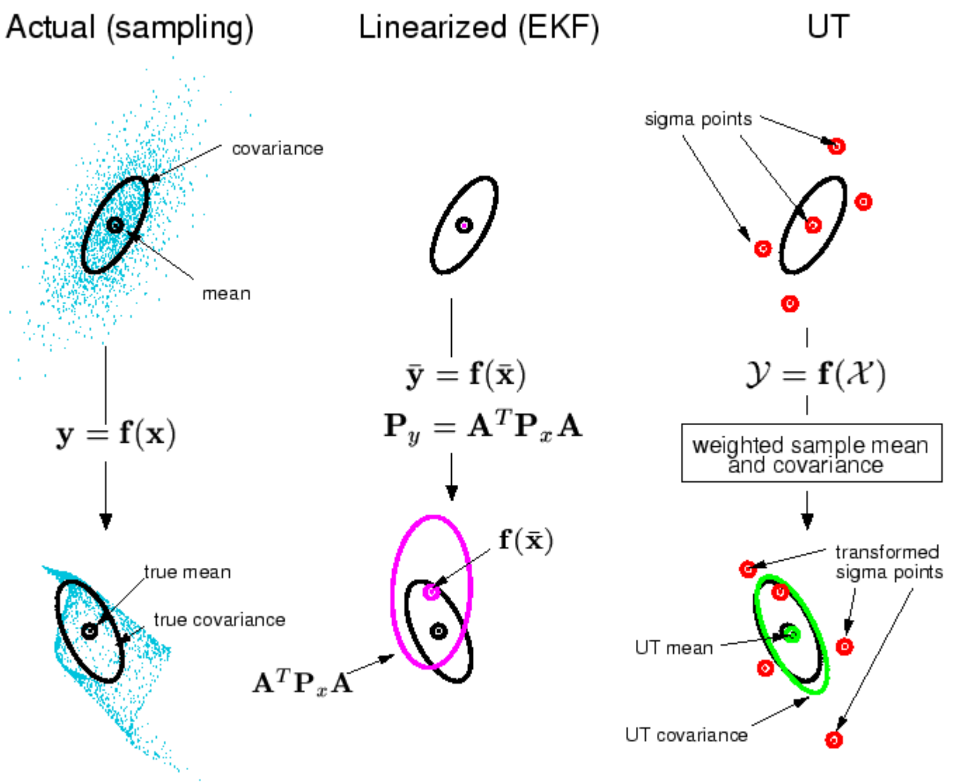
$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)\tilde{y}(k+1)$$

5.Covariance matrix of state vector

$$P(k+1|k+1) = [I - K(k)H(k)]P(k+1|k)$$

"Unscented“ Kalman filter (UKF)

A set of discretely sampled point are used to parametrise mean and covariance of distributions, avoiding linearization step. This approach is not restricted to assuming that the distributions of noise sources are Gaussian.



UKF

Initialization step

$$\hat{x}_0 = E[x_0]$$

$$P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$$

$$\hat{x}_0^a = E[x^a] = [x_0^T \; 0 \; 0]^T$$

$$P_0^a = E[(x_0^a - \hat{x}_0^a)(x_0^a - \hat{x}_0^a)^T] = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_n \end{bmatrix}$$

Sigma points

$$\hat{X}_{k-1}^a = [\hat{x}_{k-1}^a \quad \hat{x}_{k-1}^a + \gamma\sqrt{(L+\lambda)P_{k-1}^a} \quad \hat{x}_{k-1}^a - \gamma\sqrt{(L+\lambda)P_{k-1}^a}\,]$$

UKF

Time Update

$$\mathcal{X}^{x}_{k|k-1} = F[\mathcal{X}^{x}_{k-1}, u_{k-1}, \mathcal{X}^{v}_{k-1}]$$

$$\hat{x}^{-}_{k} = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{X}^{x}_{i,k|k-1}$$

$$P^{-}_{k} = \sum_{i=0}^{2L} W_i^{(c)} [\mathcal{X}^{x}_{i,k|k-1} - \hat{x}^{-}_{k}][\mathcal{X}^{x}_{i,k|k-1} - \hat{x}^{-}_{k}]^T$$

$$\mathcal{Y}_{k|k-1} = H[\mathcal{X}^{x}_{k|k-1}, \mathcal{X}^{n}_{k-1}]$$

$$\hat{y}^{-}_{k} = \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Y}_{i,k|k-1}$$

UKF

Measurement Update

$$P_{\tilde{y}_k\tilde{y}_k} = \sum_{i=0}^{2L} W_i^{(c)} \left[Y_{i,k|k-1} - \hat{y}_k^-\right]\left[Y_{i,k|k-1} - \hat{y}_k^-\right]^T$$

$$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^{(c)} \left[X_{i,k|k-1} - \hat{x}_k^-\right] \left[Y_{i,k|k-1} - \hat{y}_k^-\right]^T$$

$$K = P_{x_k y_k} P_{\tilde{y}_k\tilde{y}_k}^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - \hat{y}_k^-)$$

$$P_k = P_k^- - K_k P_{\tilde{y}_k\tilde{y}_k} K_k^T$$

където:

$$x^a = [x^T \;\; v^T \;\; n^T]^T,$$

$$X^a = [(X^x)^T \;\; (X^v)^T \;\; (X^n)^T]^T,$$

$$\gamma = \sqrt{(L + \lambda)}$$

Interactive Multiple Model - IMM

IMM algorithm models complicated non-linear system behaviour with more than one model.

$$x(k+1) = F(M(k+1))x(k) + v(k, M(k+1))$$

$$z(k+1) = H\big(M(k)\big)x(k) + w(k+1, M(k+1))$$

$$\mu_l(k) = \mu(M(k)) = P\{M(k) = l \mid z(1), z(2), \dots, z(k-1)\}$$

$$\sum_{l=1}^{r} \mu_l(k) = 1$$

IMM

$$\mu_{l|m}(k) = \mu(M(k-1) \mid M(k)) = P\{M(k-1) = l \mid M(k) = m, z(1), z(2), \ldots, z(k-1)\} =$$

$$= \frac{1}{\alpha_m} P\{M(k) = m \mid M(k-1) = l, z(1), z(2), \ldots, z(k-1)\} P(M(k-1) = l \mid, z(1), z(2), \ldots, z(k-1)),$$

$$\hat{x}_{0l}(k \mid k) = \sum_{i=1}^{r} \hat{x}_l(k) \mu_{i|l}(k)$$

$$P_{0l}(k \mid k) = \sum_{i=1}^{r} \mu_{i|l}(k \mid k) \left\{ P_i(k) + [\hat{x}_i(k) - \hat{x}_{0l}(k)][\hat{x}_i(k) - \hat{x}_{0l}(k)]' \right\}$$

$$\Lambda_l(k) = p\left[z(k) \mid M(k) = l, \hat{x}_{0l}(k-1 \mid k-1), P_{0l}(k-1 \mid k-1)\right]$$

$$\mu_l(k) = \frac{1}{c} \Lambda_l(k) \sum_{i=1}^{r} p_{il} \mu_i(k-1)$$

IMM

$$\widehat{x}(k \mid k) = \sum_{i=1}^{r} \widehat{x}_i(k) \mu_i(k)$$

$$P(k \mid k) = \sum_{i=1}^{r} \mu_i(k) \left\{ P_i(k) + [\widehat{x}_i(k) - \widehat{x}(k)][\widehat{x}_i(k) - \widehat{x}(k)]' \right\}$$

Interlaced” Extended Kalman Filter – IEKF

The fundamental idea of the IEKF is to linearise nonlinear system by means of an appropriate partition of the state space variables. Let assume we succeed separate p parts. We could start p parallel KF implementations, each one devoted to estimate only a subset of the state variable, while considering the remaining parts as deterministic time varying parameters.

IEKF

$$x(k+1) = A(k)x(k) + f(x(k), u(k)) + v(k)$$

$$z(k) = C(k)x(k) + w(k),$$

$$\begin{bmatrix} x^{(1)}(k+1) \\ x^{(2)}(k+1) \end{bmatrix} = \begin{bmatrix} F^{(1)}(k) & 0 \\ 0 & F^{(2)}(k) \end{bmatrix} \begin{bmatrix} x^{(1)}(k) \\ x^{(2)}(k) \end{bmatrix} + \begin{bmatrix} f^{(1)}(x^{(2)}(k), u(k)) \\ f^{(2)}(x^{(1)}(k), u(k)) \end{bmatrix} + \begin{bmatrix} v^{(1)}(k) \\ v^{(2)}(k) \end{bmatrix}$$

$$z(k) = C^{(1)}\left(x^{(2)}(k)\right)x^{(1)}(k) + D^{(1)}\left(x^{(2)}(k)\right) + w(k)$$

$$z(k) = C^{(2)}\left(x^{(1)}(k)\right)x^{(2)}(k) + D^{(2)}\left(x^{(1)}(k)\right) + w(k)$$

IEKF

$$\tilde{Q}^{(i)}(k) = Q^{(i)}(k) + J_{x,j}^{f,i} P^{(j)}(k-1|k-1) {J_{x,j}^{f,i}}^T + J_u^{f,i} Q^u(k) {J_u^{f,i}}^T$$

$$P^{(i)}(k|k-1) = A^{(i)} P^{(i)}(k-1|k-1) {A^{(i)}}^T + \tilde{Q}^{(i)}(k)$$

$$\tilde{R}^{(i)}(k) = R^{(i)}(k) + C^{(j)}\ (\hat{x}^{(i)}(k|k-1) P^{(j)}(k|k-1) C^{(j)}\ (\hat{x}^{(i)}(k|k-1)^T$$

$$K^{(i)}(k) = \frac{P^{(i)}(k|k-1) C^{(i)}\ (\hat{x}^{(j)}(k|k-1)^T}{C^{(i)}\ (\hat{x}^{(j)}(k|k-1) P^{(i)}(k|k-1) C^{(i)}\ (\hat{x}^{(j)}(k|k-1)^T + \tilde{R}^{(i)}(k)}$$

$$\hat{x}^{(i)}(k|k) = \hat{x}^{(i)}(k|k-1) + K^{(i)}(k)\Big[z(k) - C^{(i)}\left(\hat{x}^{(j)}(k|k-1)\right)\hat{x}^{(i)}(k|k-1) + {} + D^{(i)}\left(\hat{x}^{(j)}(k|k-1)\right)\Big]$$

$$\hat{x}^{(i)}(k+1|k) = A^{(i)}\hat{x}^{(i)}(k|k) + f^{(i)}\left(\hat{x}^{(j)}(k|k-1), u(k)\right)$$

$$P^{(i)}(k|k) = P^{(i)}(k|k-1) - K^{(i)}(k) C^{(i)}\left(\hat{x}^{(j)}(k|k-1)\right) P^{(i)}(k|k-1)$$

i = 1,2

Literature

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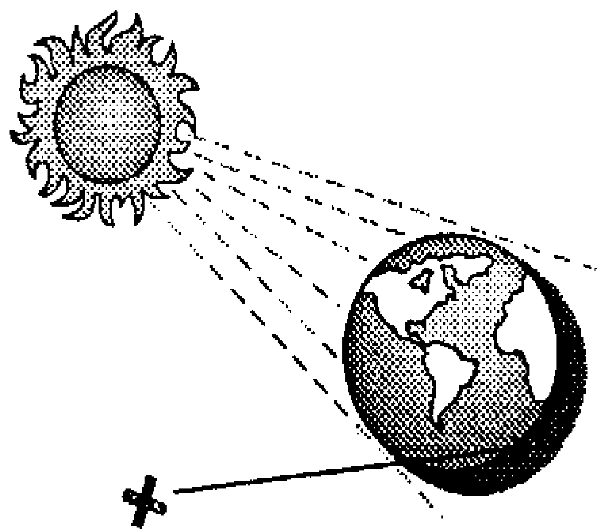
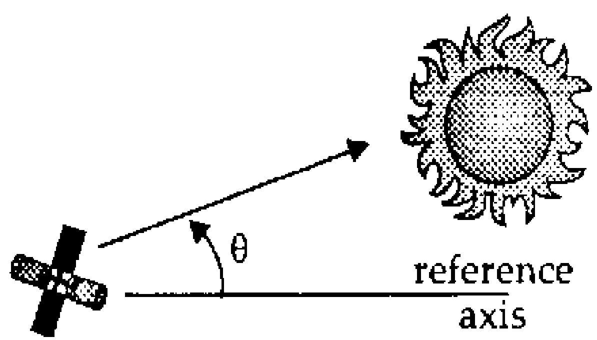
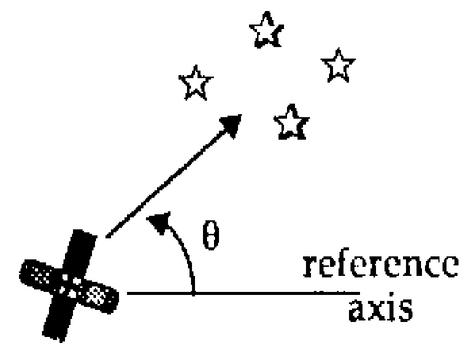
Another aided systems for attitude determination



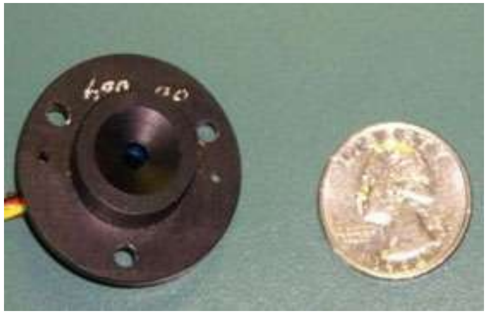
Earth Horizon Sensor



Sun Sensor

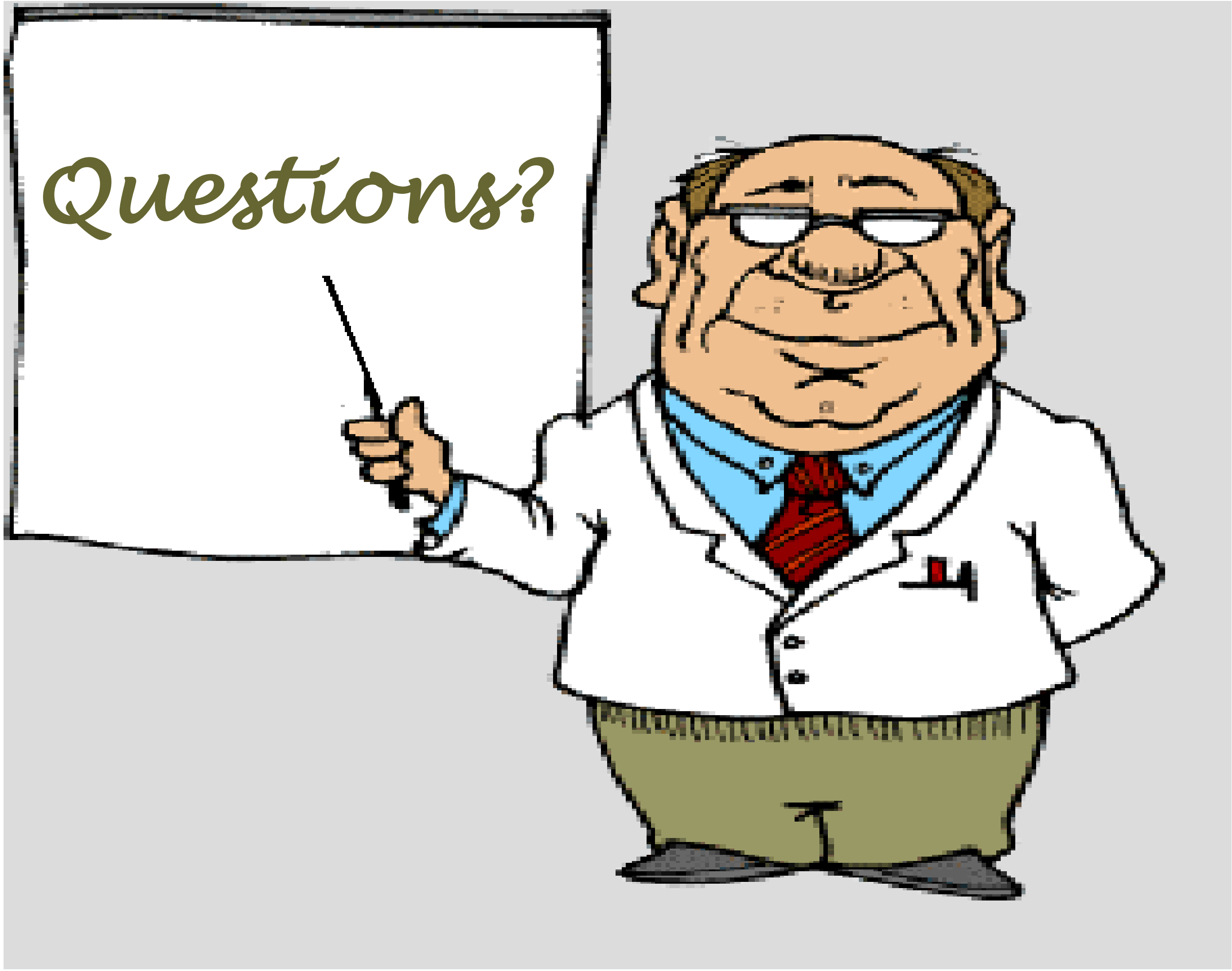


Star Tracker



Attitude determination

Sensor	Accuracies	Comments
IMU	Drift 0.0003-1 deg/h 0.001 deg/h nominal	Requires updates
Star sensor	1 arcsec-1 arcmin 0.0003-0.001 deg	2 axis for single star Multiple stars for map
Sun sensor	0.005-3 deg 0.01 deg nominal	Eclipse
Earth sensor GEO LEO	 <0.1-0.25 deg 0.1-1 deg	2 axis
Magnetometer	0.5-3 deg	<6000 km Difficult for high i


Questions?